Weighted linear dynamic logic

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Linear Temporal Logic (*LTL*), Pnueli 1977

- *LTL* $=$ *FO* logic.
- Satisfiability, validity, logical implication of *LTL* formulas: PSPACE-complete.
- *LTL*: reasonable for practical applications.
- *LTL* $\subsetneq$ finite automata.
- *LTL* $\subsetneq$ finite automata over infinite words.

Monadic Second Order (*MSO*) logic

- Büchi 1960, Elgot 1961, Trakhtenbrot 1962:

  MSO logic $=$ finite automata.

- Büchi 1962: *MSO* logic $=$ finite automata over infinite words.

- *MSO* logic formulas $\xrightarrow{\text{non-elementary}}$ finite automata.

- *MSO* logic: not reasonable for practical applications.

New logic?

- A *logic* combining the complexity properties of reasoning on *LTL* and the expressive equivalence to finite automata was greatly desirable.
- Vardi and Wolper 1994: *ETL* a Temporal logic with Automata Connectives.
- Satisfiability of *ETL*(=*RETL*) formulas is PSPACE-complete.
- Vardi 2000: *ForSpec*, industrial temporal logic used by Intel: *RETL+hardware features (clocks and resets).*
- 2003 *PSL* an industrial-standard property-specification language: *LTL* extended with dynamic modalities (borrowed from Dynamic Logic), clocks and resets.
- Vardi 2011, De Giacomo and Vardi 2013, 2015: **Linear dynamic logic** (*LDL*).
- *LDL* is a combination of propositional dynamic logic and *LTL*.

Quantitative logics required for modern applications

- Droste and Gastin 2005, 2009: Weighted *MSO* logic over semirings.
- Weighted automata $\subsetneq$ weighted *MSO* logic.
- Restricted weighted *MSO* logic $=$ weighted automata (Büchi type theorem) but the translation is non-elementary.
- Kupferman and Lustig 2007: Weighted *LTL* over De Morgan Algebras.
- Droste and Vogler 2012: Weighted *LTL* over arbitrary bounded lattices.
- Bouyer, Markey and Matteplackel 2014, Almagor, Boker and Kupferman 2014, 2016: Weighted *LTL* over $[0, 1]$.
- Mandrali and Rahonis 2014, 2016: Weighted *LTL* over semirings.
- In this paper: Weighted *LTL* over the naturals is incomparable to weighted automata.

Notations

A alphabet

$w = w(0) \ldots w(n-1) \in A^*$, with $w(i) \in A$, $0 \leq i \leq n-1$

$w_{\geq i} = w(i) \ldots w(n-1)$ for $0 \leq i \leq n-1$

LDL - Syntax

- Atomic propositions: $P = \{p_a \mid a \in A\}$.

Definition

Syntax of *LDL* formulas ψ over A :

$$\psi ::= true \mid p_a \mid \neg\psi \mid \psi \wedge \psi \mid \langle\theta\rangle\,\psi$$
$$\theta ::= \phi \mid \psi? \mid \theta + \theta \mid \theta;\theta \mid \theta^{+}$$

$p_a \in P$, ϕ propositional formula over P.

LDL - Semantics

ψ *LDL* formula, $w \in A^*$. Define $w \models \psi$ inductively:

- $w \models true$,
- $w \models p_a$ iff $w(0) = a$,
- $w \models \neg\psi$ iff $w \not\models \psi$,
- $w \models \psi_1 \wedge \psi_2$ iff $w \models \psi_1$ and $w \models \psi_2$,
- $w \models \langle \phi \rangle \psi$ iff $w \models \phi$ and $w_{\geq 1} \models \psi$,
- $w \models \langle \psi_1 ? \rangle \psi_2$ iff $w \models \psi_1$ and $w \models \psi_2$,
- $w \models \langle \theta_1 + \theta_2 \rangle \psi$ iff $w \models \langle \theta_1 \rangle \psi$ or $w \models \langle \theta_2 \rangle \psi$,
- $w \models \langle \theta_1 ; \theta_2 \rangle \psi$ iff $w = uv$, $u \models \langle \theta_1 \rangle true$, and $v \models \langle \theta_2 \rangle \psi$,
- $w \models \langle \theta^+ \rangle \psi$ iff there exists n with $1 \leq n \leq |w|$ such that $w \models \langle \theta^n \rangle \psi$,
  θ^n, $n \geq 1$ is defined by $\theta^1 = \theta$ and $\theta^n = \theta^{n-1} ; \theta$ for $n > 1$.

LDL - Main results (De Giacomo and Vardi 2013)

- *LDL* formulas $=$ rational expressions.
  - Rational expressions $\xrightarrow{\text{linear}}$ *LDL* formulas .
  - *LDL* formulas $\xrightarrow[\text{exponential}]{\text{doubly}}$ rational expressions.
  - *LDL* formulas $\xrightarrow{\text{exponential}}$ finite automata.
- Satisfiability, validity, logical implication of *LDL* formulas: PSPACE-complete.

Weighted rational expressions

- $(K, +, \cdot, 0, 1)$ semiring
- *Weighted rational expressions over A and K :*

$$E ::= ka \mid E + E \mid E \cdot E \mid E^+$$

$a \in A, k \in K$

- *Generalized weighted rational expressions over A and K :*

$$E ::= ka \mid E + E \mid E \cdot E \mid E^+ \mid E \odot E.$$

$a \in A, k \in K$

- **Semantics:** $\|E\| : A^* \to K$ *rational* (*g-rational*)
  - $\|ka\| = ka$
  - $\|E_1 + E_2\| = \|E_1\| + \|E_2\|$
  - $\|E_1 \cdot E_2\| = \|E_1\| \cdot \|E_2\|$ (Cauchy product)
  - $\|E^+\| = \|E\|^+$ $(\|E\|(\varepsilon) = 0$, proper)
  - $\|E_1 \odot E_2\| = \|E_1\| \odot \|E_2\|$ (Hadamard product)

Weighted *LDL* - Syntax

Atomic propositions: $P = \{p_a \mid a \in A\}$.

Definition

Syntax of weighted *LDL* formulas φ over A and K :

$$\varphi ::= k \mid \psi \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid \langle \rho \rangle\, \varphi$$
$$\rho ::= \phi \mid \varphi? \mid \rho \oplus \rho \mid \rho \cdot \rho \mid \rho^{\oplus}$$

$k \in K$, ϕ propositional formula over P, ψ *LDL* formula.

Weighted *LDL* - Semantics

φ weighted *LDL* formula. *Semantics* $\|\varphi\| : A^* \to K$, for $w \in A^*$:

- $\|k\|(w) = k$,
- $\|\psi\|(w) = \begin{cases} 1 & \text{if } w \models \psi \\ 0 & \text{otherwise} \end{cases}$,
- $\|\varphi_1 \oplus \varphi_2\|(w) = \|\varphi_1\|(w) + \|\varphi_2\|(w)$,
- $\|\varphi_1 \otimes \varphi_2\|(w) = \|\varphi_1\|(w) \cdot \|\varphi_2\|(w)$,
- $\|\langle\phi\rangle \varphi\|(w) = \|\phi\|(w) \cdot \|\varphi\|(w_{\geq 1})$,
- $\|\langle\varphi_1 ?\rangle \varphi_2\|(w) = \|\varphi_1\|(w) \cdot \|\varphi_2\|(w)$,
- $\|\langle\rho_1 \oplus \rho_2\rangle \varphi\|(w) = \|\langle\rho_1\rangle \varphi\|(w) + \|\langle\rho_2\rangle \varphi\|(w)$,
- $\|\langle\rho_1 \cdot \rho_2\rangle \varphi\|(w) = \sum_{w=uv} \left(\|\langle\rho_1\rangle \mathit{true}\|(u) \cdot \|\langle\rho_2\rangle \varphi\|(v)\right)$,
- $\|\langle\rho^{\oplus}\rangle \varphi\|(w) = \sum_{n\geq 1} \|\langle\rho^n\rangle \varphi\|(w)$ ($\|\langle\rho\rangle \mathit{true}\|$ proper)

  ρ^n, $n \geq 1$ is defined by $\rho^1 = \rho$ and $\rho^n = \rho^{n-1} \cdot \rho$ for $n > 1$.

Weighted *LDL* - Example

- *LDL* formula:

$$Last ::= \langle true \rangle \bigwedge_{a' \in A} \neg p_{a'}$$

$$w = w(0) \ldots w(n-1) \in A^*, \quad 0 \leq i \leq n-1$$

$$w_{\geq i} \models Last \text{ iff } w_{\geq i+1} \not\models p_{a'} \text{ for every } a' \in A \text{ iff } i = n-1$$

- $(\mathbb{N}, +, \cdot, 0, 1)$, $a \in A$, $k \in \mathbb{N} \setminus \{0\}$

$$\varphi = \left\langle (((\langle (k \otimes p_a)? \rangle \, Last)? \cdot (\langle (k \otimes p_a)? \rangle \, Last)?)^{\oplus} \right\rangle true \oplus \bigwedge_{a' \in A} \neg p_{a'}$$

- $\|\varphi\|(w) = \begin{cases} k^{2n} & \text{if } w = a^{2n}, \, n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Weighted *LDL* - Main results

- *LDL*-definable series $=$ g-rational series (No fragments for *LDL*!).
  - Weighted g-rational expressions $\xrightarrow{\text{linear}}$ weighted *LDL* formulas.
- K commutative:

  LDL-definable series $=$ rational series $=$ recognizable series.

  - Weighted *LDL* formulas $\xrightarrow[\text{exponential}]{\text{doubly}}$ weighted automata.
  - K idempotent:
    - Weighted *LDL* formulas $\xrightarrow{\text{exponential}}$ weighted automata.
- K computable field, φ, φ' weighted *LDL* formulas, $k \in K$:
  $\|\varphi\| = \|\varphi'\|$, $\|\varphi\| = \widetilde{k}$ (constant series): decidable in *doubly exponential* time.

Weighted *LDL* - Comparison to other weighted logics

- Weighted *LDL* and weighted *LTL* over the naturals are incomparable.
- Weighted *LDL* and weighted *FO* logic over the naturals are incomparable.
- *K* commutative:

$$\textbf{restricted}\text{ weighted } \mathit{MSO} \text{ logic } = \text{ weighted } \mathit{LDL},$$

$$\textbf{restricted}\text{ weighted } \mathit{FO} \text{ logic } \subsetneq \text{ weighted } \mathit{LDL},$$

$$\textbf{restricted}\text{ weighted } \mathit{LTL} \subsetneq \text{ weighted } \mathit{LDL}.$$

  - *K* dual continuous with the Arden fixed point property:

$$\text{weighted } \mathit{LDL} = \text{ weighted } \wedge\text{-free } \mu\text{-calculus.}$$

LDL over infinite words, Vardi 2011

- Modified syntax for interpretation over infinite words.

$$LDL\text{-}\omega\text{-definable languages} \;=\; \omega\text{-rational languages.}$$

- Satisfiability of *LDL* formulas: PSPACE-complete.

Weighted *LDL* over infinite words - Main results

- K totally complete semiring.
- Modified syntax for interpretation over infinite words.

$$LDL\text{-}\omega\text{-definable series} \; = \; \text{g-}\omega\text{-rational series.}$$

- No fragments for *LDL*!
- K totally commutative complete:

$$LDL\text{-}\omega\text{-definable series} \; = \; \omega\text{-rational series} \; = \; \omega\text{-recognizable series.}$$

  - K idempotent:
    - Weighted *LDL* formulas $\xrightarrow{\text{exponential}}$ weighted Büchi automata.

Weighted *LDL* over infinite words - Comparison to other weighted logics

- K totally commutative complete:

$$\textbf{restricted} \text{ weighted } \omega\text{-}MSO \text{ logic } = \text{ weighted } \omega\text{-}LDL,$$

$$\textbf{restricted} \text{ weighted } \omega\text{-}FO \text{ logic } \subsetneq \text{ weighted } \omega\text{-}LDL,$$

$$\textbf{restricted} \text{ weighted } \omega\text{-}LTL \subsetneq \text{ weighted } \omega\text{-}LDL.$$

- K dual continuous semiring with the Arden fixed point property:

$$\text{weighted } \omega\text{-}LDL = \text{ weighted } \omega\text{-} \wedge \text{-free } \mu\text{-calculus}.$$

Future research

- Translation of weighted *LDL* formulas over infinite words to weighted automata (not idempotent semirings).
    - Complexity results.
- Weighted *LDL* over more general structures, reasonable for practical applications, e.g. valuation monoids.

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Thank you

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