Window parity games: an alternative approach toward parity games with time bounds

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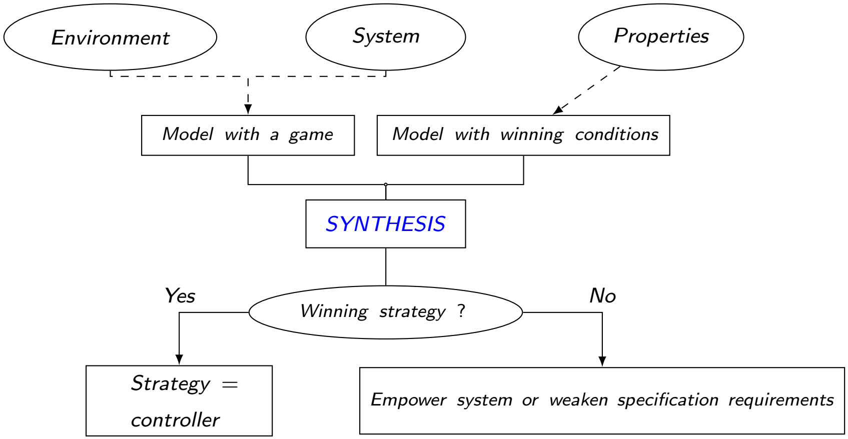
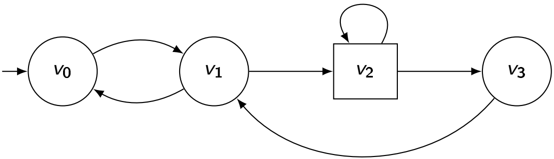
Synthesis via Game Theory

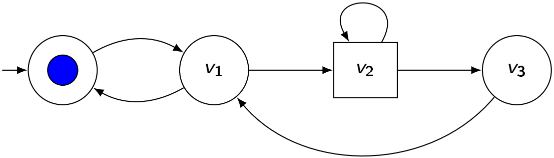


Games on graphs

- Model the antagonistic interaction between the system/player 1 (○) and the environment/player 2(□).



- Vertices and edges.



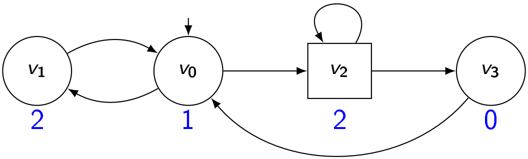
Questions

Given a game structure G, an objective Ω and an initial vertex v_0,

- Decision problem.
- Complexity of the decision problem.
- Memory requirements for winnings strategies.

- Let $p : V \to \{0, \ldots, k\}$ be a priority function.
  Parity objective : minimum priority seen infinitely often is even.

Player 1 has a memoryless winning strategy to ensure the Parity objective.



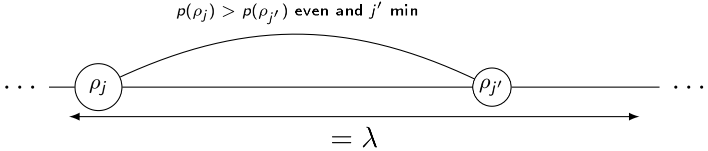
Some properties

- The decision problem is in $UP \cap coUP$ [Jur98].
- Memoryless strategies are sufficient for both players.
- Drawbacks:
  - Is there a polynomial time algorithm to solve these games ?
  - Parity objective deals with limit behavior.

    $\leadsto$ No explicit bound.
- Two approaches:
  - Parity-Response (PR) based on finitary parity [CHH09],
  - Window Parity (WP) based on window mean-payoff [CDRR15].

- Parity-response ($PR(\lambda, p)$):

Idea: given a bound λ, every odd priority has to be followed by a smaller even priority in $\lambda - 1$ steps.

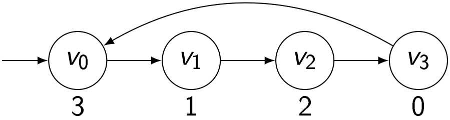


Objectives based on Parity-Response

- Fixed (Fix) objective : bound λ is given as a parameter.
- Bounded (Bnd) objective : looking for the existence of such a bound.
- For these objectives, two variants: Direct (Dir) and prefix-independent.
- Under approximations of parity objective

Example

Player 1 is winning for DirFixPR$(3, p)$: two steps ($= \lambda - 1$) to see a smaller even priority



Known results: one dimension

	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
[1] Fixed PR	PSPACE-c.	exponential	$\leq$ exponential
[2]Bounded PR	P-easy.	memoryless	infinite

$\rightsquigarrow$ PSPACE-completeness is a bad news.

[1][WZ16] : A. Weinert and M. Zimmermann. Easy to win, hard to master: Optimal strategies in parity games with costs.

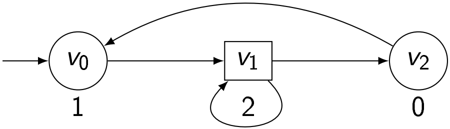
[2][CHH09]: K. Chatterjee, T. Henzinger, and F. Horn. Finitary winning in omega-regular games.

Example (1/2)

Natural question about (Dir)BndPR(p)

memoryless winning strategy for Parity ? $\Longrightarrow$? bound $\lambda = |V|$

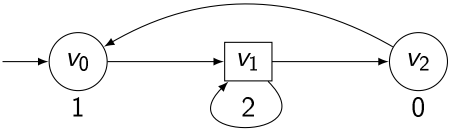
NO!!



Strategy: loop long enough on v_1

Example (2/2)

Need infinite memory for player 2 for $\Omega = \mathrm{BndPR}(p)$



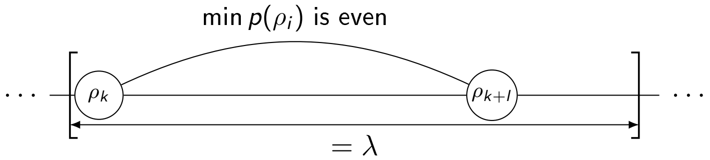
Strategy: loop longer and longer on v_1

New results

- The problem for Fixed PR is in P when λ is fixed.
- The problem for Fixed PR is in P when the biggest priority is fixed.
- Study of multidimensional setting: EXPTIME-complete, player 1 needs now exponential memory (example later).

Window Parity

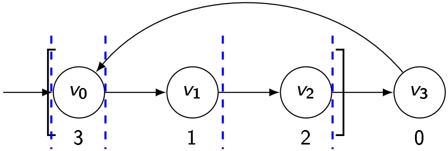
Same idea as done for Window Mean-Payoff objective [CDRR15]!



Idea: min of priorities has to be even before the end of the window.

Again, we consider Fixed and Bounded objectives + Direct and prefix-independent variants.

Example



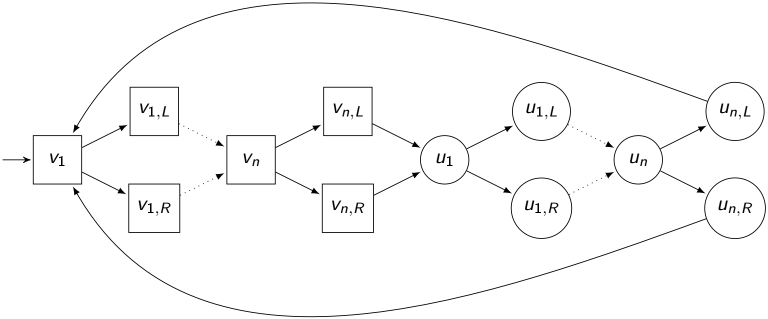
- $\rho \notin \text{FixWP}(\lambda = 3, p)$.

- In general $\text{FixWP}(\lambda, p) \neq \text{FixPR}(\lambda, p)$.
  Idea: For PR, another request may occur in the meanwhile.

Results

- Approximation of Parity objective.
- Fixed PR can be under and over approximated by Fixed WP.
- Bounded WP and Bounded PR coincide.
- Fixed WP games can be solved in polynomial time
  Idea: Keep track of the current minimum priority. If it is even, slide the window, otherwise go to next vertex if the end of the window is not reached.
- Multi-dimension: EXPTIME-complete, exponential memory for player 1.

Example: exponential memory



	one-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
Fixed WP	**P-c.**	**polynomial**	
Fixed PR	PSPACE-c.	exponential	$\leq$ exponential
Bounded WP	P-c.	memoryless	infinite
Bounded PR			

	multi-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
Fixed WP	**EXPTIME**-c.	**exponential**	
Fixed PR			
Bounded WP		**exponential**	**infinite**
Bounded PR			

Future work: logical fragments corresponding to our framework ?

Full paper available on ArXiV: `https://arxiv.org/abs/1606.01831`

Thank you!

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