Multi-Buffer Simulations for Trace Language Inclusion

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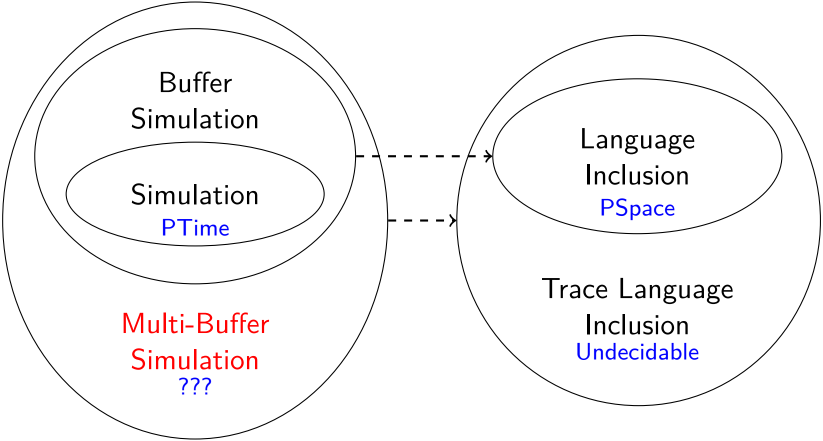
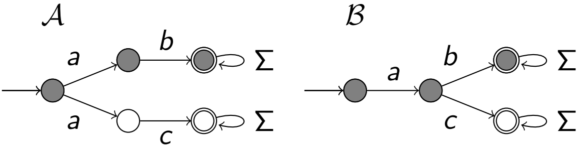
GandALF 2016, Catania
14 September 2016

Overview

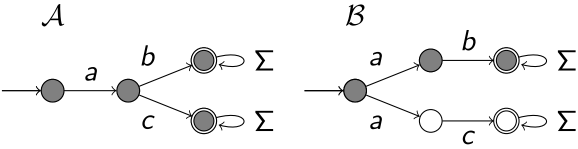

Buffer
Simulation
Simulation
PTime
Multi-Buffer
Simulation
???
Language
Inclusion
PSpace
Trace Language
Inclusion
Undecidable

Simulation $\rightsquigarrow$ Language Inclusion

- simulation approximates language inclusion: $L(\mathcal{A}) \subseteq L(\mathcal{B})$
- played between Spoiler (**S**) and Duplicator (**D**)
- construct two runs $\rho_{\mathcal{A}}$, $\rho_{\mathcal{B}}$ stepwise
- **D** wins iff $\rho_{\mathcal{B}}$ accepting or $\rho_{\mathcal{A}}$ not accepting
- $\mathcal{A} \sqsubseteq \mathcal{B}$ iff **D** has winning strategy



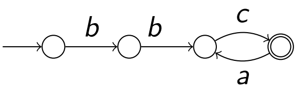
$\mathcal{A} \sqsubseteq \mathcal{B}$



$\mathcal{A} \not\sqsubseteq \mathcal{B}$

Mazurkiewicz Trace

- set of words, but some letters are allowed to commute
- equivalence class $[w]_I$, $I \subseteq \Sigma \times \Sigma$ independence relations
- Def. : $\sim_I$ is the least congruence on Σ^ω with $uabv \sim_I ubav$ for all $(a, b) \in I$ and $uv \in \Sigma^\omega$
- Ex. : $\Sigma = \{a, b, c\}$, $I = \{(a, c), (c, a), (b, c), (c, b)\}$
  $bbcaca \ldots \sim_I bcbaca \ldots \sim_I cbbaca \ldots$
  $[bb(ca)^\omega]_I = \{bbc(ac)^\omega, bcb(ac)^\omega, cbb(ac)^\omega, bb(ac)^\omega, \ldots\}$
- Def. $[L(\mathcal{A})]_I = \{u \mid u \sim_I w \text{ and } w \in L(\mathcal{A})\}$
- Ex.



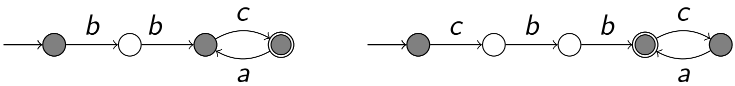
  $[L(\mathcal{A})]_I = [bb(ca)^\omega]_I$
- Given: $\mathcal{A}$, $\mathcal{B}$, and I
  Question: Is $[L(\mathcal{A})]_I \subseteq [L(\mathcal{B})]_I$? undecidable [Sakarovitch'92]
- Extend buffer simulation to approximate $L(\mathcal{A}) \subseteq [L(\mathcal{B})]_I$

Multi-Buffer Simulation

- played with m FIFO buffers on NBA $\mathcal{A}, \mathcal{B}$ over Σ
- letter distribution $\hat{\Sigma} = (\Sigma_1, \dots, \Sigma_m)$, $\Sigma_i \subseteq \Sigma$
- buffers capacities $\kappa = (k_1, \dots, k_m)$, $k_i \in \mathbb{N} \cup \{\omega\}$
- conf. : $(p, w_1, \dots, w_m, q)$, $w_i \in \Sigma_i^{\leq k_i}$
- in position $(p, w_1, \dots, w_m, q)$
  1. **S**: $p \xrightarrow{a \in \Sigma} p'$
  2. **D**: $q \xrightarrow{u \in \Sigma^*} q'$, s.t. $(aw_i)_{\Sigma_i} = (w_i' u)_{\Sigma_i}$, for all $i \in \{1, \dots, m\}$
- next position $(p', w_1', \dots, w_m', q')$
- **D** wins iff
  - $\rho_{\mathcal{A}}$ not accepting, or
  - $\rho_{\mathcal{B}}$ accepting and $|w_{\mathcal{A}}|_a = |w_{\mathcal{B}}|_a$ for all $a \in \Sigma$.
- $\mathcal{A} \sqsubseteq^{\kappa}_{\hat{\Sigma}} \mathcal{B}$ iff **D** has winning strategy

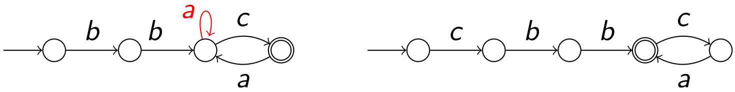
Examples

- $\hat{\Sigma} = (\{a, b\}, \{a\}, \{b\}, \{c\}) \quad \Leftrightarrow I = \{ (a, c), (c, a), (b, c), (c, b)\}$



Buff 1	b~~b~~	b~~b~~		a~~a~~		a~~a~~		...
Buff 2				a~~a~~		a~~a~~		...
Buff 3	b~~b~~	b~~b~~						...
Buff 4			c~~c~~		c~~c~~		c~~c~~	...

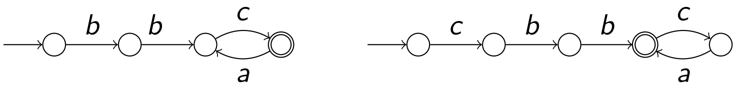
- $\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{2,1,2,0} \mathcal{B}$ But, $\mathcal{A} \not\sqsubseteq_{\hat{\Sigma}}^{2,0,2,0} \mathcal{B}$



- $\mathcal{A}' \sqsubseteq_{\hat{\Sigma}}^{\omega,\omega,2,0} \mathcal{B}$

Properties of Multi-Buffer Simulation

- Thm. Given: $\mathcal{A}$, $\mathcal{B}$, $k_1, \ldots, k_n \in \mathbb{N}$, $\hat{\Sigma}$
  Question: Is $\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{k_1,\ldots,k_n} \mathcal{B}$? decidable in PTime
  - reduce to $\mathcal{A} \sqsubseteq \mathcal{B}'$, states in $\mathcal{B}'$ are o.t.f $(q^{\mathcal{B}}, w_1, \ldots, w_n)$
- Thm. $\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{\kappa} \mathcal{B} \Rightarrow L(\mathcal{A}) \subseteq [L(\mathcal{B})]_I$
  - $\hat{\Sigma} \Leftrightarrow I$
  - Ex. : $I = \{(a,c),(c,a),(b,c),(c,b)\}$
    then $\hat{\Sigma} = (\{a,b\},\{a\},\{b\},\{c\})$



$$\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{2,1,2,0} \mathcal{B} \;\Rightarrow\; L(A) \subseteq [L(B)]_I$$

- Thm. If $k_1 \leq \ell_1, \ldots, k_n \leq \ell_n$, then and $\exists i$, $k_i < \ell_i$, then

$$\sqsubseteq^{k_1,\ldots,k_n} \subsetneq \; \sqsubseteq^{\ell_1,\ldots,\ell_n}$$

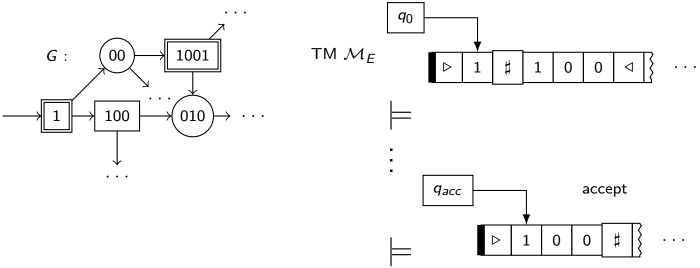
- Incremental approximation for $L(A) \subseteq [L(B)]_I$

Decidability of $\sqsubseteq_{\hat{\Sigma}}^{k_1,\ldots,k_n}$, $k_1, \ldots, k_n \in \mathbb{N} \cup \{\omega\}$

- Thm. Given: $\mathcal{A}$, $\mathcal{B}$
  Question: Is $\mathcal{A} \sqsubseteq^{\omega} \mathcal{B}$? decidable. EXPTIME-complete
  [H./Lange/Lozes'13]
- Thm. Given: $\mathcal{A}$, $\mathcal{B}$
  Question: Is $\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{k_1,\ldots,k_n} \mathcal{B}$, $k_1, \ldots, k_n \in \mathbb{N} \cup \{\omega\}$? undecidable
  Question: Is $\mathcal{A} \sqsubseteq_{\hat{\Sigma}}^{\omega,0} \mathcal{B}$? highly undecidable

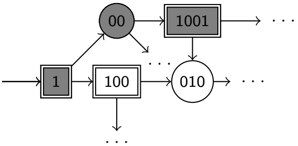
$\sqsubseteq^{\omega,0}$ is Highly Undecidable

- Thm. Deciding $\mathcal{A} \sqsubseteq^{\omega,0}_{\hat{\Sigma}} \mathcal{B}$ is $\mathbb{B}\Sigma^1_1$-hard
- Reduction from **Recursive Büchi Game** (RBG)
  Given: Computable graph G
  Question: Does **D** wins Büchi Game on G?

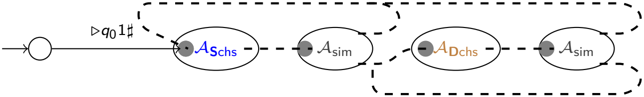


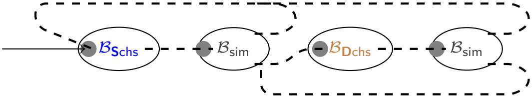
Reduction from RBG to Multi-Buffer Simulation

- Given RBG on comp. graph G



- $\rightsquigarrow$ construct $\mathcal{A}, \mathcal{B}$: (sketch)

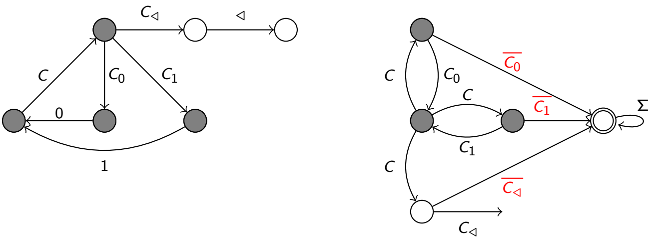




Buff1	$\triangleright$	q_0	10 1	$\sharp 0$	$0 \sharp 0$	01	$\triangleleft 0 \sharp$	0	1	$\triangleleft$
Buff2										

$\mathcal{A}_{\mathbf{D}chs}, \mathcal{B}_{\mathbf{D}chs}$

- **D** can chooses succesor node:



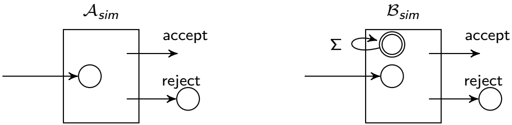
- $\Sigma_1 = \{\triangleleft, 0, 1\}$ (ω-buffer)
- $\Sigma_2 = \{C_\triangleleft, C_0, C_1, C\}$ (0-buffer)
- Ex. suppose **D** want to push 0 1 1 to ω-buffer

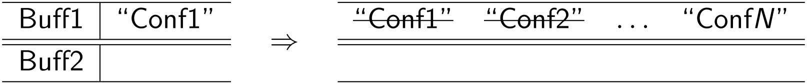
Buff1	$\triangleright$	q_0	1	0	$\sharp$	0	1	1	$\triangleleft$
Buff2									

Simulate computation of TM $\mathcal{M}$ with $\mathcal{A}_{sim}, \mathcal{B}_{sim}$

- Derive from $\delta \rightsquigarrow \hat{\delta} : (\Gamma \cup Q)^4 \to (\Gamma \cup Q)^{\leq 5}$
- Ex. $\triangleright q_0 1 \sharp 01 \triangleleft \models \triangleright \mathbf{1 q_1 \sharp 01} \triangleleft$

  $\hat{\delta}(\triangleright q_0 1 \sharp)\hat{\delta}(q_0 1 \sharp 0)\ \hat{\delta}(1 \sharp 01)\hat{\delta}(1 \sharp 01 \triangleleft) = \triangleright \mathbf{1 q_1 \sharp 01} \triangleleft$
- Encode $\hat{\delta}$ on $\mathcal{B}_{sim}$





Conf1 $\models$ Conf2 $\models \ldots \models$ Conf$N$

Conclusion and Further Work

- Multi-buffer simulation:
  - incrementally approximates trace inclusion
  - with bounded buffers is decidable in PTime
  - with unbounded buffer is highly undecidable, i.e. $\mathbb{B}\Sigma_1^1$-hard
- Further work: flush variant

Literature

- Mazurkiewicz, A. Introduction to trace theory. The Book of Traces 1995.
- Hutagalung, M. Lange, M. Lozes, E. Buffered simulation games for Büchi automata. AFL 2014.
- Sakarovitch, L. The “last” decision problem for rational trace languages. LATIN 1992.
- D. L. Dill, A. J. Hu, and H. Wong-Toi. Checking for language inclusion using simulation relations. CAV 1992.