Relation-Changing Logics as Fragments of Hybrid Logics

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  - Change the accessibility relation of a model while we are evaluating a formula.

Logics that can change the model

What about a swapping modal operator?

What happens when you add that to the basic modal logic?

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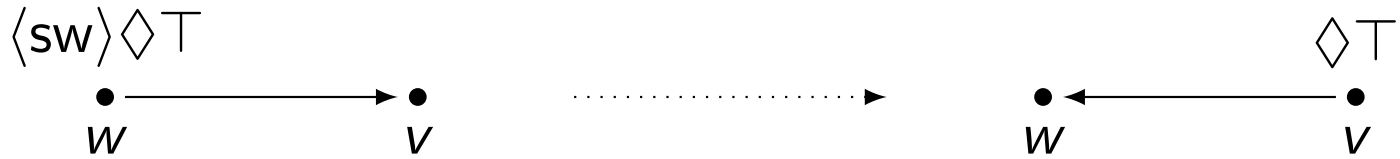

What about

- an edge-deleting modality?
- an edge-adding modality?

Sabotage Modal Logic [van Benthem 05]

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where $\mathcal{M}^{-}_{\{(u,v)\}}$ is $\mathcal{M}$ without the edge (u, v).

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We are interested in operators that can modify the accessibility relation of a model.

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We call this family of logics Relation-Changing Logics.

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We now provide translations into hybrid logics.

Hybrid Logics

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- And we also consider the down-arrow binder operator $\downarrow$:

$$\langle W, R, V \rangle, w \models \downarrow n.\varphi \quad \text{iff} \quad \langle W, R, V_n^w \rangle, w \models \varphi,$$

where $V_n^w(n) = \{w\}$ and $V_n^w(m) = V(m)$, when $n \neq m$.

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Sabotage to Hybrid Logic

We define the translation $(\ \ )'_S$ from formulas of $\mathcal{ML}(\langle \text{sb} \rangle)$ to formulas of $\mathcal{HL}(:, \downarrow)$ as:

$$(\Diamond\varphi)'_S = \downarrow n.\Diamond(\bigwedge_{xy \in S} \neg(y \wedge n{:}x) \wedge (\varphi)'_S)$$

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The translations for Bridge and Swap follow similar ideas.

Comparing Expressive Power

Theorem

$\mathcal{ML}(\blacklozenge_1) < \mathcal{HL}(:, \downarrow)$, *for* $\blacklozenge_1 \in \{\langle \mathsf{sb} \rangle, \langle \mathsf{sw} \rangle\}$.

$\mathcal{ML}(\blacklozenge_2) < \mathcal{HL}(\mathsf{E}, \downarrow)$, *for* $\blacklozenge_2 \in \{\langle \mathsf{gsb} \rangle, \langle \mathsf{gsw} \rangle, \langle \mathsf{br} \rangle, \langle \mathsf{gbr} \rangle\}$.

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Proof.

- To prove that $\mathcal{ML}(\blacklozenge_1) < \mathcal{HL}(:, \downarrow)$ it suffices to find two $\blacklozenge_1$-bisimilar models distinguishable by $\mathcal{HL}(:, \downarrow)$
- To prove that $\mathcal{ML}(\blacklozenge_2) < \mathcal{HL}(\mathsf{E}, \downarrow)$ it suffices to find two $\blacklozenge_2$-bisimilar models distinguishable by $\mathcal{HL}(\mathsf{E}, \downarrow)$

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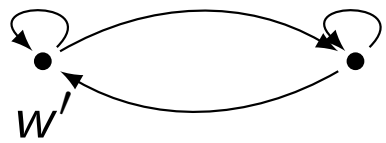
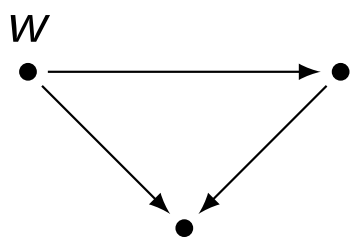
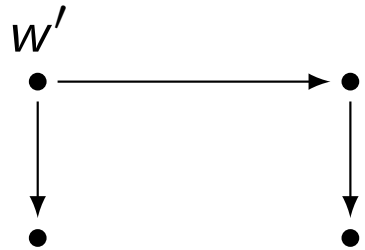
$\mathcal{M}, w$	$\mathcal{M}', w'$	Bisimilar for
		$\mathcal{ML}(\langle \mathrm{sw} \rangle)$ $\mathcal{ML}(\langle \mathrm{br} \rangle)$ $\mathcal{ML}(\langle \mathrm{gsw} \rangle)$ $\mathcal{ML}(\langle \mathrm{gbr} \rangle)$
		$\mathcal{ML}(\langle \mathrm{sb} \rangle)$ $\mathcal{ML}(\langle \mathrm{gsb} \rangle)$

The formula $\downarrow n.\Box n$ can distinguish the models in the first row.

The formula $\downarrow n.\Diamond\downarrow m.n{:}\Diamond\Diamond m$ can distinguish the models in the second row.

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Since the translations preserve equivalence, we get:

1. The satisfiability problem for $\mathcal{ML}(\langle \mathsf{sb} \rangle)$ and $\mathcal{ML}(\langle \mathsf{sw} \rangle)$ over models of bounded width is decidable.
2. The satisfiability problem for all relation-changing logics over linear, transitive trees, and $S5$ frames is decidable.

Decidable Fragments (Syntactic Restrictions)

$\mathcal{HL}(:,\downarrow) \setminus \Box\downarrow\Box$ is the fragment obtained by removing formulas that contain a nesting of $\Box$, $\downarrow$, and again $\Box$. This fragment is known to be decidable [B. ten Cate & M. Franceschet 05].

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Let $\blacklozenge \in \{\langle \mathsf{sb} \rangle, \langle \mathsf{sw} \rangle\}$ and $\blacksquare \in \{[\mathsf{sb}], [\mathsf{sw}]\}$. The following patterns are produced by the translations:

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The following fragments are decidable on the class of all models:

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2. $\mathcal{ML}(\langle \mathsf{sw} \rangle) \setminus \{\blacksquare\blacksquare, \blacksquare\Box, \Box\blacksquare, \blacksquare\blacklozenge\blacksquare\}$

where $\blacksquare$ is either $\Box$ or $\blacksquare$.

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- This implementation is useful to check the correctness of the translations and for checking that RC formulas build models in the expected way.

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