Cycle Detection in Computation-Tree Logic

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Everybody knows Model Checking

How to check system correctness.

- System represented as Mathematical Structure $\mathcal{K}$ (e.g., Kripke structure, Labeled transition system);
- Desired behavior represented as Logic Formula φ (e.g., Modal Logic, LTL, CTL, CTL*);
- The systems meets the behavior if (and only if) $\mathcal{K} \models \varphi$.

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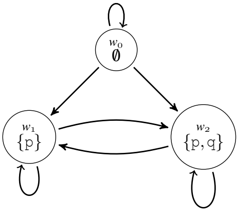
Here: we introduce and investigate a temporal logic that can explicitly express the existence (or non existence!) of a cycle.

Kripke structures

A Kripke structure is a tuple $\mathcal{K} = \langle \mathrm{AP}, \mathrm{W}, \mathrm{R}, \mathsf{L}, w_I \rangle$ with:



- $\mathrm{AP} = \{\mathtt{p}, \mathtt{q}\}$;
- $\mathrm{W} = \{w_0, w_1, w_2\}$;
- $\mathrm{R} \subseteq \mathrm{W} \times \mathrm{W}$;
- $\mathsf{L} : \mathrm{W} \to 2^{\mathrm{AP}}$;
- $w_I \in \mathrm{W}$.

Basic notions

Path

A path in $\mathcal{K}$ is an infinite sequence $w_0, w_1, \ldots$ such that $(w_i, w_{i+1}) \in \mathrm{R}$, for all $i \in \mathbb{N}$.

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Cycle

A cycle in $\mathcal{K}$ is a path $w_0, w_1, \ldots$ such that w_0 occurs infinitely many times.

Outline

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1. Computation-Tree Logic with Cycle Detection
2. Model-theoretic properties
3. Decision problems
4. Conclusions and future works

Computation-Tree Logic with Cycle Detection

Syntax

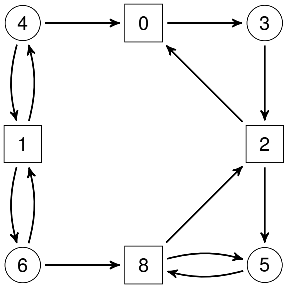
$\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \mathrm{E}\psi \mid \mathrm{A}\psi \mid \mathrm{E}^{\circlearrowleft}\psi \mid \mathrm{A}^{\circlearrowleft}\psi$
$\psi := \phi \mid \neg\psi \mid \psi \wedge \psi \mid \psi \vee \psi \mid \mathrm{X}\psi \mid \psi \mathrm{U} \psi$

Two new path quantifiers $\mathrm{E}^{\circlearrowleft}$ and $\mathrm{A}^{\circlearrowleft}$, predicating over cycles.

Semantics

- $\mathcal{K}, w \models \mathrm{E}^{\circlearrowleft}\psi$ if there exists a cycle π starting from w such that $\mathcal{K}, \pi \models \psi$;
- $\mathcal{K}, w \models \mathrm{A}^{\circlearrowleft}\psi$ if, for all cycles π starting from w, it holds that $\mathcal{K}, \pi \models \psi$.

Example (Parity Games)



- V_0: nodes of player 0;
- V_1: nodes of player 1;
- $E \subseteq (V_0 \cup V_1) \times (V_0 \cup V_1)$;
- $p : (V_0 \cup V_1) \to \mathbb{N}$;
- An outcome of the game is a (infinite) path π determined by moves of the players in their respective nodes.

π is winning for Player 0 if the biggest priority occurring infinitely often is even.
π is prompt winning for Player 0 if every occurrence of an infinitely occurent odd number k is followed by a bigger even number n with bounded delay.

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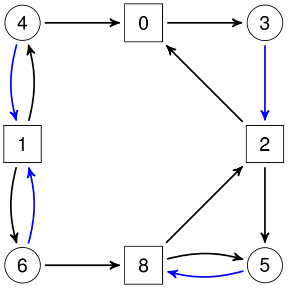

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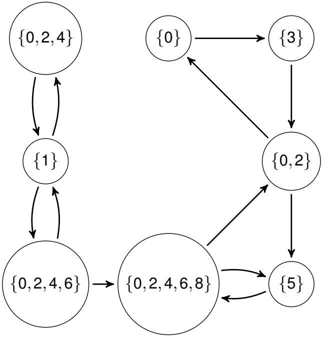
A strategy for Player 0 is a function mapping each node in V_0 to an outgoing arrow.
A strategy is winning if it enforces only winning paths.
A strategy is prompt winning if it enforces only prompt winning paths.

Strategy projection



- Strategy projection for Player 0 produces a Kripke structure;
- The paths in this structure are the ones enforced by Player 0;
- We can use $\mathrm{CTL}^{\star}$ and $\mathrm{CTL}^{\star}_{\circlearrowright}$ to represent properties of the strategy in the game.

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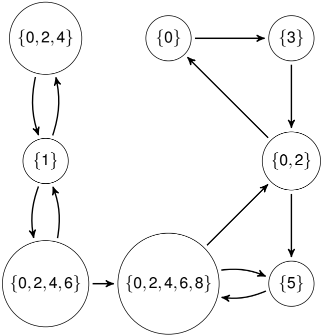

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$$\varphi^{par} = \mathtt{A}\bigwedge_{k\equiv_2 1}(\mathtt{GF}k \rightarrow \mathtt{GF}(k+1))$$

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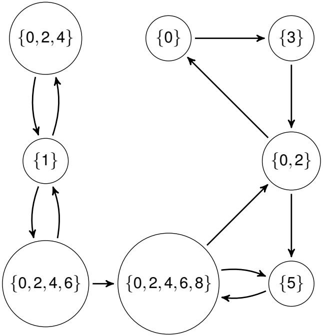

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Prompt winning strategy

$$\varphi^{par} \wedge \neg (\bigvee_{n \equiv_2 0} \mathtt{E} (\bigvee_{k<n, k \equiv_2 1} (\mathtt{GF}\, k \wedge \mathtt{G}(k \rightarrow \mathtt{G}\neg(K+1) \mathtt{U} \mathtt{E}^{\circlearrowleft} \mathtt{G} \neg (k+1)))))$$

Outline

Bisimulation?

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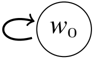

Figure: A Kripke structure $\mathcal{K}$

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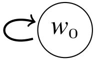

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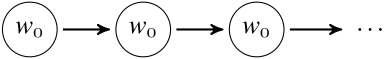

Figure: Tree unwinding $\mathcal{T}_{\mathcal{K}}$

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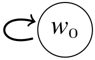

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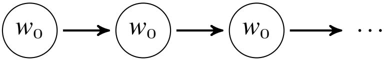

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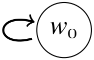

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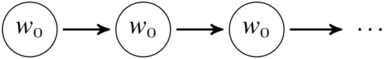

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$\mathcal{T}_{\mathcal{K}}, w_0 \not\models \mathrm{E}^{\circlearrowright}\top$

Theorem

- *$CTL^{\star}_{\circlearrowright}$ is not invariant under tree-unwinding*
- *$CTL^{\star}_{\circlearrowright}$ is not invariant under bisimulation*

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$CTL^{\star}_{\circlearrowleft}$ does not have the finite-model property.

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CTL* is invariant under bisimulation.

Theorem

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Proof intuition

μCalculus is invariant under bisimulation, but CTL$^{\star}_{\circlearrowleft}$ cannot express all regular expressions, e.g., the *counting-by-two*.

Cycle-Bisimulation

Definition

Cycle-Bisimulation

Two Kripke structures $\mathcal{K}_1$ and $\mathcal{K}_2$ are cycle-bisimilar if there exists a bisimulation relation $\mathrm{B} \subseteq \mathrm{W}_{\mathcal{K}_1} \times \mathrm{W}_{\mathcal{K}_2}$ on their states such that, for all $(w_1, w_2) \in \mathrm{B}$:

- For every cycle π_1 starting from w_1 there exists a cycle π_2 starting from w_2 bisimilar to π_1 *state-by-state*;
- For every cycle π_2 starting from w_2 there exists a cycle π_1 starting from w_1 bisimilar to π_2 *state-by-state*.

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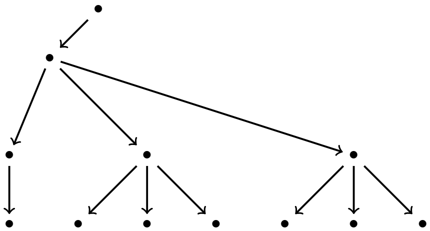
$CTL^{\star}_{\circlearrowleft}$ is invariant under cycle-bisimulation.

Tree with Back Edges



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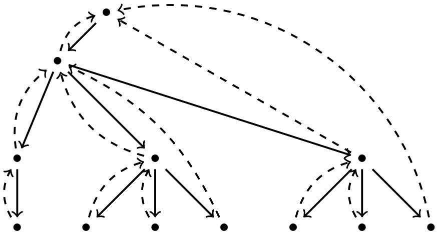

- A simple tree $\mathcal{T} = (\mathrm{V}, \mathsf{E})$
- Plus a partial mapping f from nodes to *back edges* such that:
  - $\mathsf{f}(w)$ is an ancestor of w;
  - Back edges do not overlap.

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For every Kripke structure $\mathcal{K}$, there exists a tree with back edges $\mathcal{T}_{\mathcal{K}}$ which is cycle-bisimilar.

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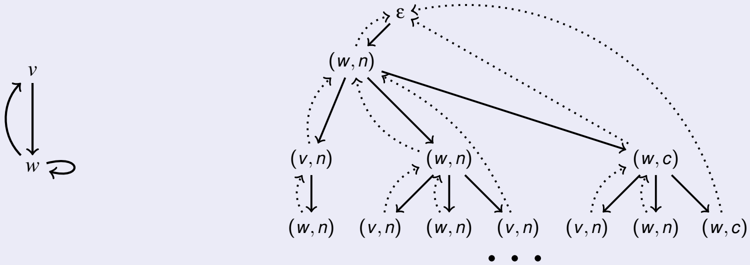
An example



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Proof idea

Bottom-up automata-theoretic technique construction borrowed from $\mathrm{CTL}^{\star}$.
The cases $\mathtt{E}^{\circlearrowleft}\psi$ and $\mathtt{A}^{\circlearrowleft}\psi$ are similar to $\mathtt{E}\psi$ and $\mathtt{A}\psi$ with an additional check that the initial state occurs infinitely often. It suffices to intersect the automaton $\mathcal{N}_{\mathcal{K}}^{\mathtt{E}\psi}$ with a suitably defined Büchi word automaton for this additional check.

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Exploiting the tree-like model property and cycle-bisimulation invariance we can build a two-way parity tree automaton $\mathcal{A}_\varphi$ that:

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The satisfiability problem for CTL$^\star_\circlearrowleft$ can be solved in 3ExpTime and it is 2ExpTime-Hard.

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- Compared its expressive power with well known formalisms like $\mathrm{CTL}^{\star}$ and μCalculus ;
- Investigated on its model properties like invariance under bisimulation;
- Addressed both Model-Checking and Satisfiability problems.

Future works

- Fill the gap in complexity for the satisfiability problem;
- Further syntactic extension in which the cycle symbol is atomic (work in progress);
- Quantifying over cycles in logics for open systems (e.g., $\mathsf{ATL}^{\star}$, and SL).

Thank you!